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Fragmentation production of doubly heavy baryons

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Abstract

Baryons with a single heavy quark are being studied experimentally at present. Baryons with two units of heavy flavor will be abundantly produced not only at future colliders, but also at existing facilities. In this paper we study the production via heavy quark fragmentation of baryons containing two heavy quarks at the Tevatron, the LHC, HERA, and the NLC. The production rate is woefully small at HERA and at the NLC, but significant at pp and $p\bar{p}$ machines. We present distributions in various kinematical variables in addition to the integrated cross sections at hadron colliders.

The constituent quark model has been remarkably successful in describing the observed hadronic states. This includes mesons with one heavy and one light quark (*i.e.*, B and D mesons), baryons with one heavy and two light quarks, and the J/ψ and Υ mesons, which contain a heavy quark and heavy antiquark. Recent experiments have made great progress in the observations of the c and b baryons[1, 2] and their decays[3, 4]. The constituent quark model predicts the existence of baryons containing two heavy quarks (cc , bc , or bb) and one light quark, and we are on the verge of obtaining the necessary experimental sensitivity to observe these states. The energies necessary to produce these particles are already reached; the difficulty remaining is in their reconstruction. These states have in general a large number of decay modes, so that their observation and a measurement of their properties will require a large number of them to be produced. This difficulty is increased by the fact that the production rates at e^+e^- colliders are extremely small, so that the identification of these particles must take place in the messier environment of hadronic collisions.

Were the heavy quark of infinite mass, the two heavy quarks would be bound in a point-like diquark, and the light degrees of freedom would “see” this diquark as a static antitriplet color source. In this limit, the interactions of the light degrees of freedom with the heavy diquark are quite similar to interactions of the light degrees of freedom with the \bar{b} quark in a B meson[5, 6]. For realistic heavy quark masses, this simple picture is not yet valid. In particular, the hyperfine splittings of these baryons are not yet well described by the heavy-mass limit[7]. The states may nonetheless be accurately treated as a combination of light degrees of freedom and heavy although not-pointlike diquark. The interactions between the two heavy quarks are analogous to the $Q\bar{Q}$ system familiar from J/ψ and Υ spectroscopy. At short distances, the interaction will be dominated by one-gluon-exchange, although with a color factor $2/3$ from the QQ interaction rather than $4/3$ from $Q\bar{Q}$, and at long distances it will be confining[8].

The production of these states is reliably calculable as a hard process. One takes the full set of Feynman diagrams for the production of two pairs of heavy quarks and heavy antiquarks and requires that the momenta of the two heavy quarks be nearly equal, then projects out the correct quantum numbers for the relevant Q_1Q_2 state. This is similar to the

calculation of the hard production of heavy quarkonium states[9], though more complicated as an additional $Q\bar{Q}$ pair is needed. On the other hand, because the quarks are heavy, fragmentation functions for a heavy quark to produce a doubly heavy diquark can be predicted in perturbative QCD[10, 11]. The fragmentation description of the Q_1Q_2 production process is not valid where the masses of the heavy quarks become important, that is, for small p_T or small energies, but for high-energy colliders such as LEP or Tevatron it should be applicable. The calculation is similar to that described in [11] for J/ψ production. The form of the matrix element is different in the two cases, requiring a somewhat changed method of projection onto the proper spin states. For the production of a $Q\bar{Q}$ bound state, the standard procedure is to replace the heavy quark production matrix element $\bar{u}\mathcal{O}v = \text{Tr}(\mathcal{O}v\bar{u})$ with the trace $\text{Tr}(\mathcal{O}P_{SS_z})$, where P_{SS_z} is the proper projection onto the spin of the $Q\bar{Q}$ state[12]. This procedure is, however, not simply applicable to the Q_1Q_2 bound states due to the form of the matrix element, which contains not u and \bar{v} as in the production of $Q\bar{Q}$ but u_1 and u_2 . The projection procedure described above is nothing more than a clever method of performing the sum over the quark spins. This sum can also be performed by summing over helicity amplitudes with the proper Clebsch-Gordan coefficients. Although the procedure is somewhat different, the calculations are identical in their essentials, and the fragmentation function is simply proportional to that for a $Q\bar{Q}$ pair. The differences reflect the changed color and statistical factors. The fragmentation function for Q_1 to produce a spin-1 diquark (Q_1Q_2) is

$$D_{Q_1 \rightarrow (Q_1Q_2)}(z, \mu = (m_1 + 2m_2)) = \frac{2N_{12}}{9\pi} \frac{|R_{(Q_1Q_2)}(0)|^2}{m_2^3} \alpha_s^2(2m_2) F(z), \quad (1)$$

$$F(z) = \frac{rz(1-z)^2}{4(1-z+rz)^6} \left[2 - 2z(3-2r) + 3z^2(3-2r+4r^2) - 2z^3(1-r)(4-r+2r^2) - z^4(1-r)^2(3-2r+2r^2) \right], \quad (2)$$

where $r = m_2/(m_1 + m_2)$. The factor $N_{12} = (1)2$ for (un)equal quark masses, and reflects the presence of identical fermions in the final state. For production of a spin-0 state, $(Q_1Q_2)'$,

the function $F(z)$ must be replaced by

$$F'(z) = \frac{rz(1-z)^2}{12(1-z+rz)^6} \left[6 - 18z(1-2r) + z^2(21-74r+68r^2) - 2z^3(1-r)(6-19r+18r^2) + 3z^4(1-r)^2(1-2r+2r^2) \right]. \quad (3)$$

As in the case of hard quarkonium production, the requirement of nearly equal momenta and the projection onto the proper Q_1Q_2 quantum numbers will modify the p_T spectrum of the diquark; it will fall more rapidly than that of single heavy quark production. For heavy quarkonium production, the faster fall-off in p_T of the hard production process means that fragmentation production will dominate for sufficiently large p_T despite the suppression of the fragmentation production by powers of α_s [13]. For the Q_1Q_2 states, no hard production processes of lower order in α_s exist. We assume that the doubly heavy diquarks will always hadronize into baryons with two heavy quarks, so that the fragmentation function $D_{Q_1 \rightarrow \text{baryon}} = D_{Q_1 \rightarrow (Q_1Q_2)}$.

A Q_1Q_2 diquark in the ground state has a symmetric spatial wavefunction. Because the Q_1Q_2 must combine with a light quark to produce a colorless baryon, the diquark must be a color antitriplet (antisymmetric) state. For identical quarks the wavefunction must satisfy Fermi-Dirac statistics, and thus the cc and bb ground state diquarks have spin 1. The bc states are not restricted by statistics, and we denote the spin singlet (triplet) state by bc' (bc). The spin triplet can hadronize into a spin-1/2 or -3/2 baryon, Ξ_{bc} and Ξ_{bc}^* , respectively; the spin singlet can produce only the spin-1/2 Ξ'_{bc} . The probabilities for the fragmentation of heavy quarks into these baryons are: $c \rightarrow \Xi_{cc}, \Xi_{cc}^*$, about 2×10^{-5} ; $b \rightarrow \Xi'_{bc}$, about 4×10^{-5} ; $b \rightarrow \Xi_{bc}, \Xi_{bc}^*$, about 5×10^{-5} . The remaining probabilities are suppressed by $\sim (m_c/m_b)^3$ and therefore approximately two orders of magnitude smaller[10].

In addition to these probabilities, it is desirable to have predictions of the p_T (or other kinematical variables) distributions for the production of these states. The fragmentation functions given above at low momentum scale μ_0 must be evolved to higher scales μ in order to be of use in calculations for high energy colliders:

$$d\sigma(A+B \rightarrow (Q_1Q_2) + X) = \sum_i \int_0^1 dz d\sigma(A+B \rightarrow i(\frac{p_T}{z}, \eta) + X, \mu^2) D_{i \rightarrow (Q_1Q_2)}(z, \mu^2). \quad (4)$$

The evolution occurs via an Altarelli-Parisi equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} D_{i \rightarrow (Q_1 Q_2)}(z, \mu^2) = \frac{\alpha_s}{2\pi} \sum_j \int_z^1 \frac{dy}{y} P_{ij}(z/y) D_{j \rightarrow (Q_1 Q_2)}(y, \mu^2). \quad (5)$$

which does not change the overall probabilities given above. In the evolution of the fragmentation functions via (5), we include only the P_{QQ} splitting function. The fragmentation of gluons to heavy diquarks is suppressed relative to that of heavy quarks by α_s , and may be neglected. A separate numerical evolution of the fragmentation functions to each z and μ needed in the course of a numerical simulation would be an extremely computer-intensive approach; instead we generate values of the fragmentation function for a reasonably-spaced set of μ and z values and interpolate to the desired points. The unevolved fragmentation functions depend on $|R(0)|^2$, the non-relativistic radial wavefunction at the origin, the heavy quark masses, and $\alpha_s(\mu_0)$. These input parameters are given in Table 1.

	α_s	m_Q	$ R(0) ^2$
$c \rightarrow (cc)$	0.238	1.5 GeV	$(0.65 \text{ GeV})^3$
$c \rightarrow (bc)$	0.173	4.5 GeV	$(0.80 \text{ GeV})^3$
$c \rightarrow (bc)'$	0.173	4.5 GeV	$(0.80 \text{ GeV})^3$
$b \rightarrow (bb)$	0.173	4.5 GeV	$(1.20 \text{ GeV})^3$
$b \rightarrow (bc)$	0.238	1.5 GeV	$(0.80 \text{ GeV})^3$
$b \rightarrow (bc)'$	0.238	1.5 GeV	$(0.80 \text{ GeV})^3$

Table 1: Parameters used to calculate the unevolved fragmentation functions

The J/ψ and Υ decays to lepton pairs give the wavefunctions $|R(0)|^2$ for these states relatively accurately[16] and independently of phenomenological potential models. The QCD-corrected expression for the leptonic decay of heavy quarkonium states

$$\Gamma(V \rightarrow \ell^+ \ell^-) = \frac{4\alpha^2 e_Q^2}{M_V^2} |R_V(0)|^2 \left[1 - \frac{16}{3\pi} \alpha_s(m_Q) \right] \quad (6)$$

is used to extract the Υ and J/ψ radial wavefunction at the origin. There is no information on the B_c meson to determine the wavefunction for the $b\bar{c}$ states, and the annihilation of this state into lepton pairs would not exist in any case to provide the measurement. However,

there is an empirical relationship among leptonic decays of the ϕ , J/ψ and Υ mesons, $\Gamma(V \rightarrow e^+e^-) \simeq 12e_Q^2 \text{ keV}$, which indicates that $|R(0)|^2/\mu^2$ is approximately constant.

The one-gluon exchange contribution for Q_1Q_2 differs from that for $Q_1\bar{Q}_2$ by a relative color factor 1/2. In a Coulomb potential, therefore, the relation $|R_{J/\psi}(0)|^2 = 8|R_{cc}(0)|^2$ must hold. The heavy quarks are, however, not in a pure Coulomb potential, so that we must rely on phenomenological potentials to better estimate the radial wavefunction. While many functional forms for the potential provide quite good fits to the heavy quarkonium data, the fitted potentials all have very similar shapes in the region of $r \sim 1 \text{ GeV}^{-1}$. We use 1/2 the $Q_1\bar{Q}_2$ potential for the Q_1Q_2 potential, and find the wavefunction at the origin using a numerical solution of the Schrödinger equation. We compare in Table 2 the results of this calculation for two familiar potentials with the values extracted from the J/ψ and Υ decay rates.

	Indiana[14]	Richardson[15]	data[22]
$c\bar{c}$	$(0.95 \text{ GeV})^3$	$(0.93 \text{ GeV})^3$	$(0.98 \text{ GeV})^3$
cc	$(0.67 \text{ GeV})^3$	$(0.65 \text{ GeV})^3$	
$b\bar{c}$	$(1.18 \text{ GeV})^3$	$(1.18 \text{ GeV})^3$	
bc	$(0.82 \text{ GeV})^3$	$(0.81 \text{ GeV})^3$	
$b\bar{b}$	$(1.72 \text{ GeV})^3$	$(1.88 \text{ GeV})^3$	$(1.96 \text{ GeV})^3$
bb	$(1.15 \text{ GeV})^3$	$(1.23 \text{ GeV})^3$	

Table 2: Radial wavefunctions at the origin, $|R(0)|^2$, for $Q_1\bar{Q}_2$ and Q_1Q_2 states for two interquark potentials compared to values extracted from data, see (6).

Doubly heavy baryons will be produced at all present and future accelerators where there is sufficient energy. The results presented here include both baryon and antibaryon production. We consider first the e^+e^- colliders LEP, LEP II, and NLC. At LEP, the only subprocess for the production of a heavy quark Q is $e^+e^- \rightarrow Z$, with the Z decaying to $Q\bar{Q}$. The production rates in this case are quite small, a few events per year. For LEP II and NLC the additional processes $e^+e^- \rightarrow W^+W^-$ where one W decays to a c quark and $e^+e^- \rightarrow ZZ$ where one Z decays to $c\bar{c}$ or $b\bar{b}$ also contribute (depending on the LEP II energy, of course). Despite the additional subprocesses, of which W^+W^- dominates for NLC, the event rates remain woefully small. At LEP II, only a few events/year can be expected, and a $60 \text{ fb}^{-1}/\text{yr}$,

500 GeV linear collider will produce only of order 30 events per year. The production rates at $\gamma\gamma$ colliders and $e\gamma$ colliders were also calculated, including backscattered and Weizsäcker-Williams photons and the contributions from resolved-photon processes. The cross sections for production of these particles at these colliders are also hopelessly small.

The results at HERA are similar. The main subprocesses here are $\gamma g \rightarrow Q\bar{Q}$ and $\gamma Q \rightarrow gQ$. Assuming the design luminosity of $200 \text{ pb}^{-1}/\text{yr}$, we again find that the event rate is rather small, of order 30 events/year. The resolved photon processes are likewise negligible.

The situation is considerably more hopeful at hadron colliders. The relevant subprocesses are $gg \rightarrow Q\bar{Q}$, $q\bar{q} \rightarrow Q\bar{Q}$, $gQ \rightarrow gQ$, and $qQ \rightarrow qQ$ (in the latter two subprocesses, q and Q stand for both quarks and antiquarks), although the main contribution is from the gluon-fusion subprocess. For the Tevatron, with an integrated luminosity of $100 \text{ pb}^{-1}/\text{yr}$, we anticipate of order 8×10^4 events/year ($p_T > 5 \text{ GeV}$, $|\eta| < 0.5$), while the LHC, with $100 \text{ fb}^{-1}/\text{yr}$, should produce of order 1.3×10^8 events per year ($p_T > 10 \text{ GeV}$, $|\eta| < 0.5$). Total production cross sections per unit rapidity for the various diquarks at the Tevatron and the LHC (operating at both 10 TeV and 14 TeV) are given in Table 3. The rates for production of the bc -type states are similar to those expected for the B_c meson, where rates of 10^4 B_c 's have been predicted at the Tevatron ($p_T > 10 \text{ GeV}$) and 10^7 at LHC ($p_T > 20 \text{ GeV}$)[17]. On the other hand, the production rate for cc -type states is significantly smaller than that for J/ψ production at the Tevatron which is expected to be 5×10^6 ($p_T > 5 \text{ GeV}$)[13].

	Tevatron	LHC (10 TeV)	LHC (14 TeV)
$\Xi_{cc} + \Xi_{cc}^*$	430 pb	330 pb	470 pb
Ξ'_{bc}	145 pb	220 pb	330 pb
$\Xi_{bc} + \Xi_{bc}^*$	215 pb	350 pb	490 pb
$\Xi_{bb} + \Xi_{bb}^*$	16 pb	27 pb	36 pb

Table 3: Production cross sections per unit rapidity in the central region.

Some results are given in Figs. 1 and 2. Fig. 1 shows $d\sigma/dp_T/d\eta|_{\eta=0}$ vs. p_T . A kinematical cut of 5 GeV for the baryon p_T has been imposed here. Fig. 2 gives the rapidity distribution of Q_1Q_2 baryons, again with $p_T > 5 \text{ GeV}$. The rapidity distributions are

rather flat in the central region, so that $d\sigma/dp_T$ may be estimated from $d\sigma/dp_T/d\eta|_{\eta=0}$ by multiplying with the η -region desired.

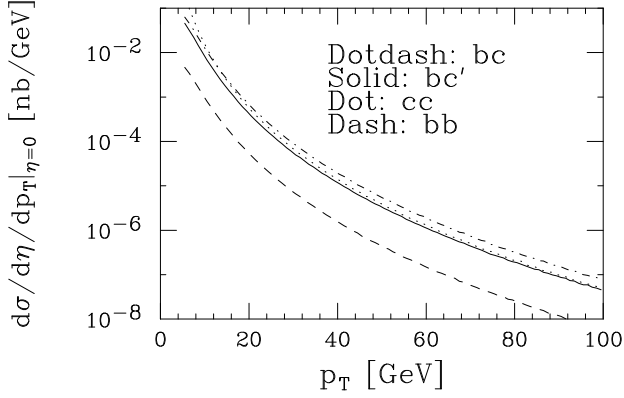


Figure 1: p_T distributions (at small rapidity) for doubly heavy baryon production at the Tevatron.

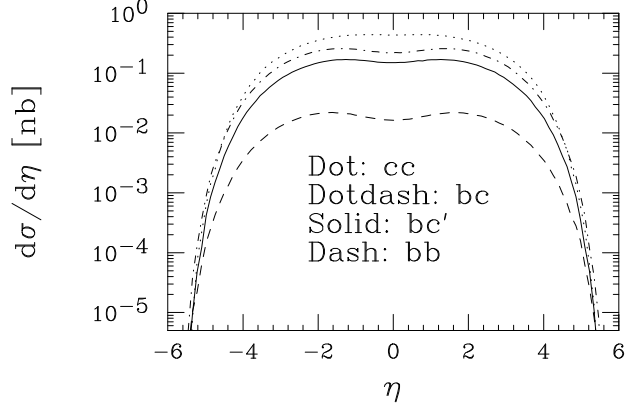


Figure 2: Pseudorapidity distributions for doubly heavy baryon production at the Tevatron.

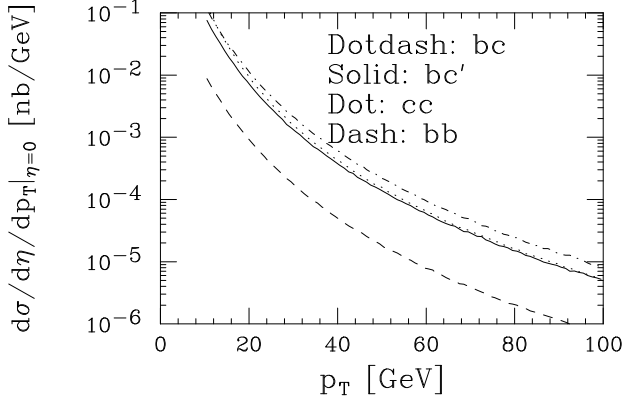


Figure 3: p_T distributions (at small rapidity) for doubly heavy baryon production at the LHC, 14 TeV.

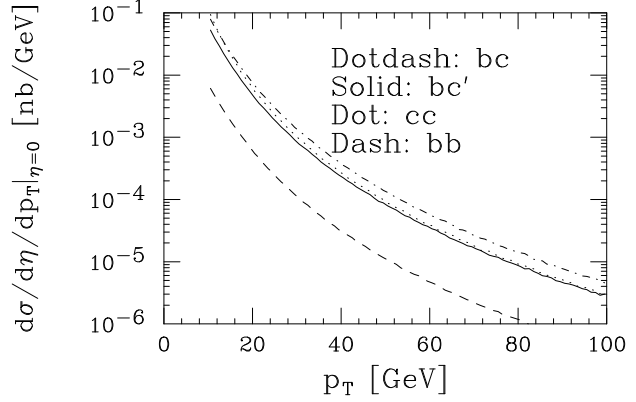


Figure 4: p_T distributions (at small rapidity) for doubly heavy baryon production at the LHC, 10 TeV.

Figs. 3 and 4 show $d\sigma/dp_T/d\eta|_{\eta=0}$ vs. p_T for the LHC operating at 14 TeV (Fig. 3) and 10 TeV (Fig. 4). A kinematical cut of $p_T > 10$ GeV is placed on the Q_1Q_2 baryon. The production rate at the lower center-of-mass energy is slightly lower, but even at the lower energy, the LHC will produce Q_1Q_2 baryons copiously. The rapidity distributions are again relatively flat in the central region ($|\eta| < 3$), and event rates for any specific detector can be estimated by simply multiplying the results in Table 3 by the η coverage of the detector.

In order to estimate the uncertainties due to the choice of factorization and fragmentation scales, we vary these scales (all chosen to be equal) by a factor of 2 from our nominal choice $\mu = p_T$. The effect of varying the scale on $d\sigma/dp_T/d\eta|_{\eta=0}$ vs. p_T can be seen in Fig. 5, and the effect on $d\sigma/d\eta$ vs. η can be seen in Fig. 6. The processes shown are representative of all the processes. The behavior seen in Fig. 6 is somewhat counter-intuitive in that one expects the rate with $\mu = p_T/2$ to be larger than that for $\mu = p_T$. However, the fragmentation functions are cut off at low μ_{frag} . This causes the drop at low p_T for this choice seen in Fig. 5 and the lower cross sections in Fig. 6. We use MRSA[18] parton distribution functions. The effect of changing the parton distribution set choice is negligible — far smaller than the effect of varying the choice of μ . Other sources of theoretical uncertainty include the parameters used for the unevolved fragmentation functions, primarily $|R(0)|^2$ where different phenomenological potentials give about 10% differences in the result, unknown QCD corrections to the parton-level cross sections, the fragmentation function and the evolution equations, and unknown relativistic corrections to the initial fragmentation functions.

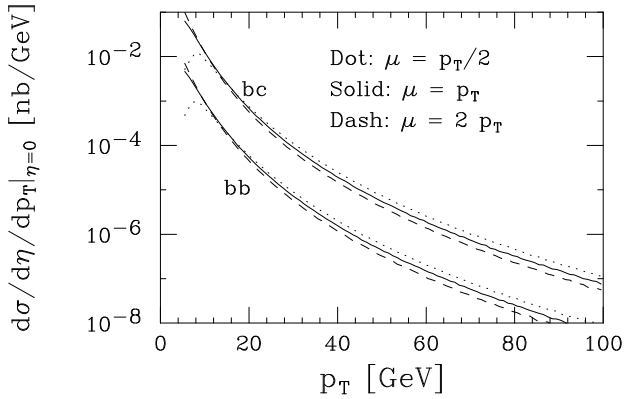


Figure 5: p_T distributions (at small rapidity) for bb and bc production at the Tevatron for different choices of scale μ .

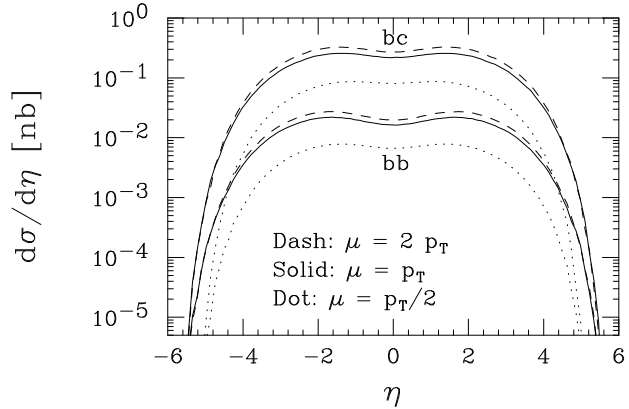


Figure 6: Pseudorapidity distributions for bb and bc production at the Tevatron for different choices of scale μ .

Recently there has been some concern that the fragmentation approximation may not work as well as anticipated for B_c production. This assertion is based on a comparison between the full calculation and the fragmentation approximation for B_c in $\gamma\gamma$ collisions[19]. There is, of course, a significant difference between $\gamma\gamma$ and gg initial states, and the convolution of the gluon distributions with the parton-level subprocess can have a large effect.

The calculation of $gg \rightarrow B_c X$ is much more complex than that of $\gamma\gamma \rightarrow B_c X$, but is necessary to understand the validity of the fragmentation approximation. The authors of [19] went on to study B_c production at hadron colliders[17]. They found that the fragmentation approximation still differs from the full calculation at the parton level ($\hat{\sigma}[gg \rightarrow b\bar{b}(\rightarrow B_c X)]$ *vs.* $\hat{\sigma}[gg \rightarrow B_c b\bar{c}]$) except at high p_T , but after the convolution of $\hat{\sigma}$ with the gluon distribution functions, the full calculation and the fragmentation approximation agree quite well for $p_T > 10$ GeV both at Tevatron and LHC energies. Fig. 6 of [17] shows that the agreement between the full calculation and the fragmentation approximation is acceptable down to $p_T^{min} = 5$ GeV at the Tevatron and $p_T^{min} = 10$ GeV at LHC. As the fragmentation functions and the production mechanisms are extremely similar for B_c and heavy diquark production, our calculation of the production of doubly heavy baryons should also be accurate in this p_T range.

The decay modes of doubly charmed baryons have been studied in [20] using SU(3) flavor symmetry. This approach relates the decays of these particles, and could be used to find relations among the decays of the bb baryons and among the bc baryons. The transitions between the doubly heavy baryons[6] and some decays of the bc baryons[21] have been discussed in the heavy quark limit. The lifetimes of c and b quarks are not very different ($\tau_{\Lambda_c} \sim 0.2 \times 10^{-12}$ s while $\tau_{\Lambda_b} \sim 1.07 \times 10^{-12}$ s[22]), so that the weak decays of both b and c are important when considering the decays of the bc baryons. bc baryons will decay weakly (both hadronically and semi-leptonically) to single b quark baryons as well as doubly charmed baryons. Both of these possibilities have been discussed in the literature[20, 23, 24]. Either a single b baryon or a doubly charmed baryon traced back to a displaced vertex using a vertex detector will be a clear sign of doubly heavy baryon production[25]. In the case of bb baryons, the production of a doubly charmed baryon (possibly with same sign di-lepton!), with both b -quark decays tagged by a vertex detector, would be an impressive signal. Unfortunately, the bb baryons are produced less frequently than the other doubly heavy baryons, and two semi-leptonic decays will severely reduce the event rate.

In conclusion, the production via fragmentation of baryons containing two heavy quarks has been calculated in the fragmentation approximation. The fragmentation approximation reproduces well the full calculation of B_c production at hadron colliders for the

p_T studied here[17]; it is expected that the fragmentation approximation will work well for doubly heavy diquark production as well. The production rates of these baryons at e^+e^- colliders and at the ep collider HERA are found to be negligible. The situation is much better at hadron colliders, with approximately 8×10^4 events/yr expected at Tevatron and 1.3×10^8 events/year at LHC. In addition to predictions for total production cross sections ($|\eta| < 0.5$ and $p_T > 5(10)$ GeV at the Tevatron (LHC)), the distributions $d\sigma/dp_T/d\eta|_{\eta=0}$ and $d\sigma/d\eta$ for $p_T > 5(10)$ GeV at the Tevatron (LHC) are studied. The detection of these particles and measurements of their properties will provide an experimental challenge due to the large number of their decay modes, but will be a rich testing ground, *e.g.*, for HQET and QCD potential models.

Acknowledgments

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